

waveguide by means of a probe has been designed for the 30-GHz band. The launcher is simple and can be an integral part of stripline circuits. The return loss is larger than 30 dB over the frequency interval from 28.5 to 33 GHz. The insertion loss of a transition over this band is 0.1 dB.

REFERENCES

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On the Definitions of Parameters in Ferrite-Electromagnetic Wave Interactions

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Abstract—Conflicting definitions of the parameters characterizing the material in ferrite-electromagnetic wave interactions occur in the current literature, as well as do actual sign errors that are difficult to detect because of the variety of signs appended to the relevant quantities by different authors. A harmonious set of definitions are set out and their consistency illustrated using the case of a uniform circularly polarized plane wave.

This letter is prompted by the fact that in the literature dealing with problems in the interaction of electromagnetic waves with ferrite media, conflicting definitions for the parameters describing the material are current. Indeed, actual sign errors, which are difficult to detect, occur in the literature.¹ Especially when one author borrows a result from another, great care in checking for consistency of sign is needed. It would be highly desirable to have a self-consistent set of equations on record in which negative signs are removed from quantities, such as frequencies, which are normally taken as positive.

Consider the system of an electron with its spin axis displaced from the direction of a superimposed dc magnetizing field H_0 (see Fig. 1). The equation of motion for a conglomerate of such elementary systems becomes, in this MKS system of units

$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mathbf{B} = -\gamma \mathbf{M} \times \mathbf{H} \quad (1)$$

where \mathbf{M} is the total magnetic moment per unit volume, \mathbf{H} the total magnetic field intensity, and the gyromagnetic ratio is given by

$$\gamma = \frac{g|e|\mu_0}{2m} \quad (2)$$

(when nuclear spin or a positron is considered, then the sign of the right-hand side of the equation of motion (1) will change accordingly. As here defined γ is an essentially positive quantity).

Taking

$$\mathbf{M} = \mathbf{M}_0 + m e^{j\omega t}, \quad \mathbf{M}_0 = M_0 \mathbf{a}_z \quad (3)$$

$$\mathbf{H} = \mathbf{H}_0 + h e^{j\omega t}, \quad \mathbf{H}_0 = H_0 \mathbf{a}_z \quad (4)$$

where M_0 and H_0 are positive quantities, and impressing a circularly polarized electromagnetic wave propagating in the direction of the dc magnetizing field of the form

$$\mathbf{e}^{\pm} = (\mathbf{a}_x \mp j\mathbf{a}_y) e^{j(\omega t - \beta z)} \quad (5)$$

$$\mathbf{h}^{\pm} = \frac{\omega \epsilon}{\beta} (\pm j\mathbf{a}_x + \mathbf{a}_y) e^{j(\omega t - \beta z)} \quad (6)$$

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¹ As an example, the classic work *Microwave Ferrites and Ferrimagnetics* by Lax and Button contains a sign error in their equation (4.5) (seemingly based on a confusion of sign of the gyromagnetic ratio which is positive in their use throughout). As a consequence all their subsequent equations in chapter 4, though self-consistent, are in error in the same way, i.e., resonance occurs for left-handed circular polarization, the same as the precession. But in chapter 7, equation (7.9) gives $h_z = +jh_y$ for the right-handed circular polarization, but the plus sign does not go with the plus in $\Gamma_{\pm} = (-\omega^2/c^2)K_{\pm}(\mu \pm \kappa)$ of eq. (7.10) if the quantities of chapter 4 are used; though subsequent usage, e.g., fig. 7.2 and accompanying equations, rather implies that the correspondence was intended. These errors are difficult to disentangle because, for the most part, they refer merely to one direction of circular polarization or its opposite, rather than to right- and left-handed. Normally, one uses \pm in this latter sense.

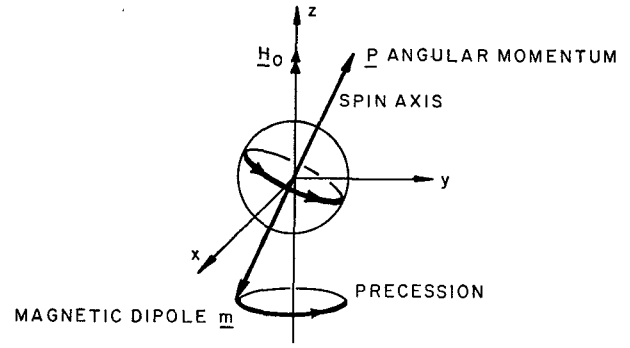


Fig. 1. Note that the letters with underbars in the figure appear bold face in the text.

the following result is obtained

$$(\beta^{\pm})^2 = \omega^2 \mu_0 \epsilon \left(1 + \frac{\omega_m}{\omega_0 \mp \omega} \right) \quad (7)$$

upon employing (1) and the Maxwellian equation

$$\nabla \times \mathbf{e} = -j\omega \mu_0 (\mathbf{h} + \mathbf{m}). \quad (8)$$

The superscripts refer to the sense of the circular polarization with the plus sign indicating right-hand polarization. The latter is here defined as being characterized by a clockwise rotation of the field vectors when viewed in the *direction of propagation*. The radian frequencies are defined by

$$\omega_0 = \gamma H_0 \quad \omega_m = \gamma M_0 \quad (9)$$

and are positive quantities.

The susceptibility matrix for the material takes the form, which is the commonest, though not a universal choice,

$$\bar{\mathbf{u}} = \mu_0 \begin{bmatrix} 1 + \chi & -j\kappa & 0 \\ +j\kappa & 1 + \chi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (10)$$

provided that χ and κ are determined by

$$\chi = \frac{\omega_0 \omega_m}{\omega_0^2 - \omega^2} \quad \kappa = -\frac{\omega \omega_m}{\omega_0^2 - \omega^2} \quad (11)$$

The propagation factors for the circularly polarized waves are then given by

$$\begin{aligned} (\beta^{\pm})^2 &= \omega^2 \mu_0 \epsilon (1 + \chi \mp \kappa) \\ &= \omega^2 \mu_0 \epsilon \left(1 + \frac{\omega_m}{\omega_0 \mp \omega} \right). \end{aligned} \quad (12)$$

Note that the $\mp \kappa$ and $\mp \omega$ occur together.

Thus the above definitions lead to consistent results with the strong interaction occurring for the circularly polarized wave that has the same rotational sense as the precession of the magnetic dipole, as it must. In addition, confusion is minimized in that the signs associated with the effective permeability when expressed in χ and κ carry over when ω_0 and ω_m are used instead.

It should also be noted that when the direction of the dc magnetizing field is reversed the positive nature of ω_0 and ω_m is retained, but the signs of ω_0 and ω_m in (11) and (12) are reversed. Hence the strong interaction will now occur, as it should, for the left-hand polarized wave.

It is perhaps worth emphasizing here that positive or right-handed polarization is defined in relation to the direction of propagation, not the applied field. This is in accordance with conventional use for plane waves, for which an applied field is irrelevant. Defining the sense of circular polarization with respect to the applied field, as is sometimes done, introduces yet a further source of sign confusion to this subject.

The above analysis is based on a time reference $e^{+j\omega t}$. Some users prefer $e^{-j\omega t}$, in which case the sign of j in the preceding results needs to be changed throughout.